1. The actual performance factors $\{\eta^{real}_{G,k-1}, \eta^{real}_{d,k-1}, d \in D\}$ are obtained from online measurements and the process mass, energy and entropy balances.
2. The differences β between model and actual values are calculated:

$$
\begin{aligned}
&\beta_{G,i,k-1} = \eta^{real}_{G,i,k-1} - p_{G,i}(\mathbf{u}_{k-1}) \qquad && i = 1...ng \\
&\beta_{d,k-1} = \eta^{real}_{d,k-1} - p_d(\mathbf{u}_{k-1}) \qquad && \forall d \in D / z_{dk-1} = 1
\end{aligned}
\tag{4}
$$

The modifiers a and $\mathbf{b}$ are obtained with a weighted linear regression, using the last nr periods and a weight w. The number of periods and the weighting strategy can be selected for each implementation, and even for each performance equation. Different criteria for choosing the weights have been proposed [13, 14]. For a disjunction d, if modifiers a_d and $\mathbf{b}_d$ only appear if $z_d = true$, they will only be updated when this disjunction was active in the last RTO cycle:

$$
\left.
\begin{aligned}
&a_{G,i,k}, \mathbf{b}_{G,i,k} = \arg\min_{a,\mathbf{b}} \sum_{j=k-nr-1}^{k-1} w_j \cdot \left(\beta_{G,i,j} - a - \mathbf{b}^T \cdot \mathbf{v}_{G,i,j}\right) \\
&\quad s.t. \qquad a^L_{G,i} \le a \le a^U_{G,i} \quad ; \quad \mathbf{b}^L_{G,i} \le \mathbf{b} \le \mathbf{b}^U_{G,i}
\end{aligned}
\right\} \quad i = 1...ng
$$

$$
\left.
\begin{aligned}
&a_{d,k}, \mathbf{b}_{d,k} = \arg\min_{a,\mathbf{b}} \sum_{j=k-nr-1}^{k-1} w_j \cdot z_{dj} \cdot \left(\beta_{dj} - a - \mathbf{b}^T \cdot \mathbf{v}_d\right) \\
&\quad s.t. \qquad a^L_d \le a \le a^U_d \quad ; \quad \mathbf{b}^L_d \le \mathbf{b} \le \mathbf{b}^U_d
\end{aligned}
\right\} \quad \forall d \in D / z_{d,k-1} = true
\tag{5}
$$

Forecast Update. At the same time, prices and demand forecasts are updated. If a new forecast is available, it can be used; otherwise, the original forecasts can be biased using the current error between actual and forecasted values.

Optimization. After updating the model and the forecasts, the real-time optimization stage is performed before the beginning of each period k. In order to reduce computational time of the problem, the full scheduling horizon is divided in smaller subsets j:

$$
\{1..N\} = \{1..k_{f1}\} \cup \{k_{f1}+1...k_{f2}\} \cup ... \cup \{k_{fT}+1...N\}
\tag{6}
$$

where k_{fj} (j=1…T) is the final period of each subset. In practice, a possible size for each subset is of 1 day (with k_{fj} the last period of the day). It allows dealing with common constraints and costs present in heat and power systems (maximum daily power consumption, total daily natural gas use, total daily emissions, among others).

The optimization problem to be solved is that of Eq. (1), with the addition of Eq. (2). The prediction of the performance factors $\{\eta_G, \eta_d, d \in D\}$ is modified as proposed in Eq.(3), using the most recent results of Eq. (5). The next period k is set as the initial period, and the closest future $k_{f,j}$ is selected as the scheduling horizon. If k is equal to the current $k_{f,j}$, a single-period optimization is performed, and the next instance of the optimizer will run with $k_{f,j+1}$ as horizon.

Figure 1 shows a diagram of the integrated SO+RTO system. In addition to Stages 1 and 2, a data validation step is proposed for correcting the measured inputs and outputs $\mathbf{u}_m^k$ and $\mathbf{y}_k^m$, respectively [15, 16]; and also a validation or filtering step for the optimal results [17, 18].

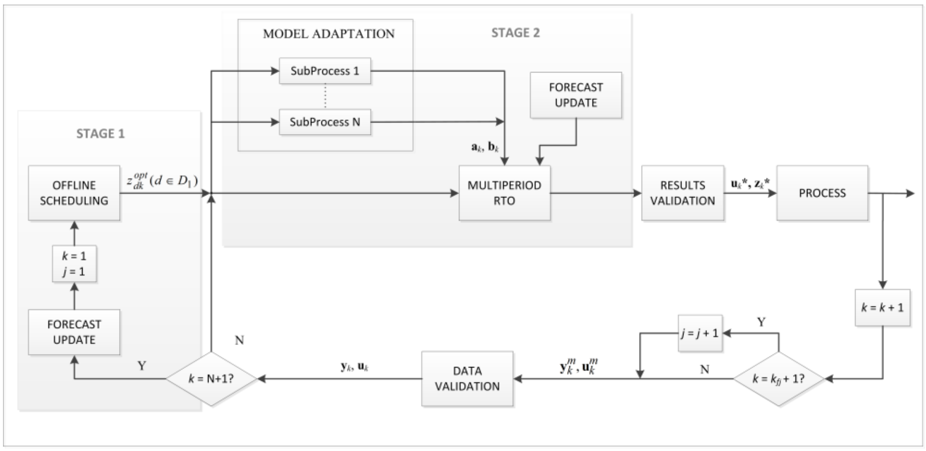

Fig. 1. Diagram of the integrated SO+RTO system.

4 Case Study: Application to a Heat and Power System

4.1 Case Description

A heat and power system (shown in Figure 2) was modeled. It consists of three boilers, a gas turbine, a heat recovery steam generator with supplementary firing (afterburning), an extraction-condensing turbine generating power, three steam turbines with spare motors for driving pumps, a steam demand from an industrial process, letdown valves and steam vents. The level of total daily NO_x emissions is constrained to be lower than 1000 kg/day. The system is studied with an horizon of 7 days; boilers startups and shutdowns are selected in Stage 1 (initial schedule), while on/off decisions for afterburning and steam turbines are selected in real-time (with a fixed horizon at the end of each day). The periods have a length of 6 hours.

All mass, energy and entropy balances were modeled rigorously. For performance factors (boilers efficiencies, gas turbine heat rate and maximum power, turbines efficiencies), as well as for NO_x emissions in boilers, gas turbine and afterburning, an approximate model is has been built. NOx emissions have been modeled using emission factors [19].

For the purpose of evaluating the proposed strategy, a different model of the performance factors was built. It is assumed to represent exactly the process under study.

This model (called hereafter *real plant*) is used to simulate the *real* process, evaluate the actual cost achieved after applying optimization solutions and provide the factors η_G^{real} required for Eq.(4). The main (structural) differences between the approximate model and the real plant are summarized in Table 1. It can be noticed that the NO_x factor for the afterburning changes with time (i.e. with the cycle k). The rest of the differences (performance of turbines, turbogenerator and gas turbine) can be obtained from reference [8].

A forecast for power price, power demand, steam demand and ambient temperature is used for scheduling and RTO. For the purpose of evaluation of the approach, a different set of demands, power price and temperature represents the *actual values* of these properties along the scheduling horizon. These actual values are used for simulation (together with the *real plant* model) and for updating the forecast for the real-time optimization model (by biasing the current and future values using the error between actual and forecasted value at current cycle k). Forecasted and actual values are shown in Figure 3.

Table 1. Differences between the approximated model and the real plant. F: Boiler steam flow (t/h); Q: Boiler/Gas Turbine/Afterburning fuel use (Gcal/h); k: Period number

Performance index	Approximate model	Real plant
Efficiency Boilers (%)	92	$75 + 0.48 \cdot F - 0.0035 \cdot F^2$
NOx Boiler 1 (kg/Gcal)	0.08	$0.07+2 \cdot Q^{-1}$
NOx Boiler 2 (kg/Gcal)	0.08	$0.08+3 \cdot Q^{-1}$
NOx Boiler 3 (kg/Gcal)	0.08	$0.06+4 \cdot Q^{-1}$
NOx Gas Turbine (kg/Gcal)	0.055	$0.03+10 \cdot Q^{-1}$
NOx Afterburning (kg/Gcal)	0.07	$0.05+5 \cdot Q^{-1}+0.001 \cdot k$

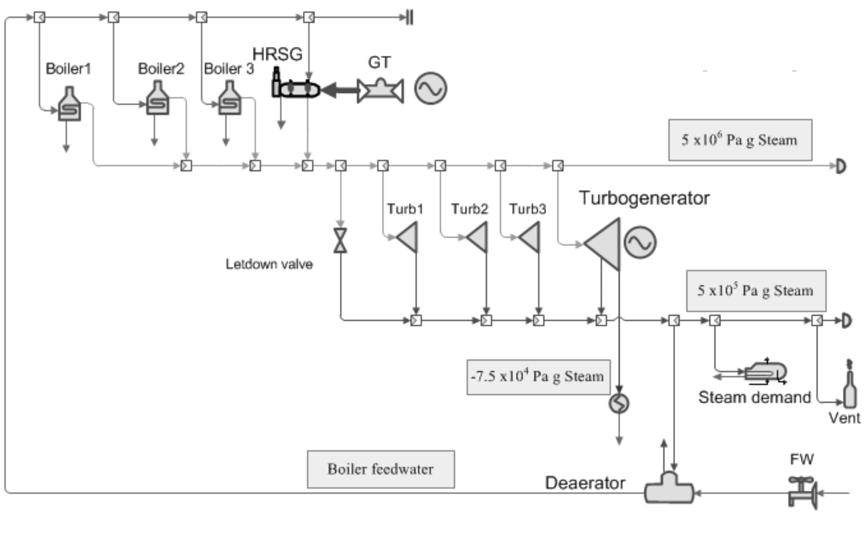

Fig. 2. Heat and power system diagram**.**

The objective function to minimize is the operating cost:

$$\sum_{k=k_0}^{k_f} \left(\begin{array}{l} 7.2 \cdot \left(Q_{TG,k} + Q_{AB,k} + \sum_{bl=1}^{3} Q_{bl,k} \right) + 0.5 \cdot FW_k + C_{pow,k} \cdot P_k \\ + 20 \cdot \sum_{t=1}^{3} \left(z_{t,k}^{on-off} + z_{t,k}^{off-on} \right) \end{array} \right) + 50. \sum_j EX_j^{NO_x} \tag{6}$$

where $Q_{TG,k}$, $Q_{AB,k}$ and $Q_{bl,k}$ are the fuel consumptions in the gas turbine, the afterburning and a boiler, respectively, in Gcal/h); FW_k is the boiler feedwater use in t/h, $C_{pow,k}$ is the power purchase cost (\$/MWh) and P_k is the net power import (MW); and $z_{t,k}^{on-off}$ and $z_{t,k}^{off-on}$ are binary variables indicating that, a turbine t has been turned off or on, respectively, in cycle k. After each day j, the excess NOx produced over the constraint $EX_j^{NO_x}$ (i.e. all NOx production higher than 1000 kg/h) is penalized with a cost.

Other constraints include the need for keeping always one boiler in operation, and that each backpressure turbine can only be turned on or off once every 24 hours. The turbogenerator cannot be stopped.

4.2 Results and Discussion

The case study was implemented in GAMS and solved using DICOPT. Stage 1 (full schedule) has 2541 equations and 2745 variables (196 discrete). It is solved in a computer with an Intel Core i7-2670QM (2.2 GHz) processor and 8 GB of RAM memory RAM. The CPU time required for the solution was of 8.7 s. Each of the real-time stages requires a CPU usage < 1.9 s, and it has a maximum size of 362 equations and 418 variables (16 discrete).

The results obtained by applying only Stage 1 are also calculated for comparison. The actual cost and constraints are calculated for each period using the *real* model.

The total operating cost for the comparison case is \$204792, while the proposed strategy leads to a cost of \$161182. The relative reduction in cost is 21.3%.

Figure 4 shows the cost evolution for the proposed strategy and the comparison case. It can be observed that the model and forecast updates provided by the real-time optimization strategy impact on a reduction of total cost in all periods. The NOx penalty cost cannot be related to each period but for each day, but for the purpose of illustrating the results it is distributed equally over the periods of the corresponding day (25% of the total daily NOx penalty is added to the cost of each period of the day).

Figure 5 shows the evolution of NOx total production. Again, the feedback properties of real-time optimization reduce the number and the size of violations of the daily

NOx production constraint. The plant-model mismatch and the uncertainty in the forecasted demands and prices lead to errors in the predicted fuel use and NOx production in the boilers and the gas turbine, which is (at least partially) corrected by real-time optimization. In this example, the model does not predict any violations of the NOx constraint for any of the days. However, the application of the results of the comparison case to the real plant causes a NOx production higher than the allowed limit. In this case, the limit is modeled as a "soft" constraint, which can be slightly violated but with a high penalization in the objective function. In other situations, a limit like this could be a hard constraint, and therefore the proposed schedule would be infeasible.

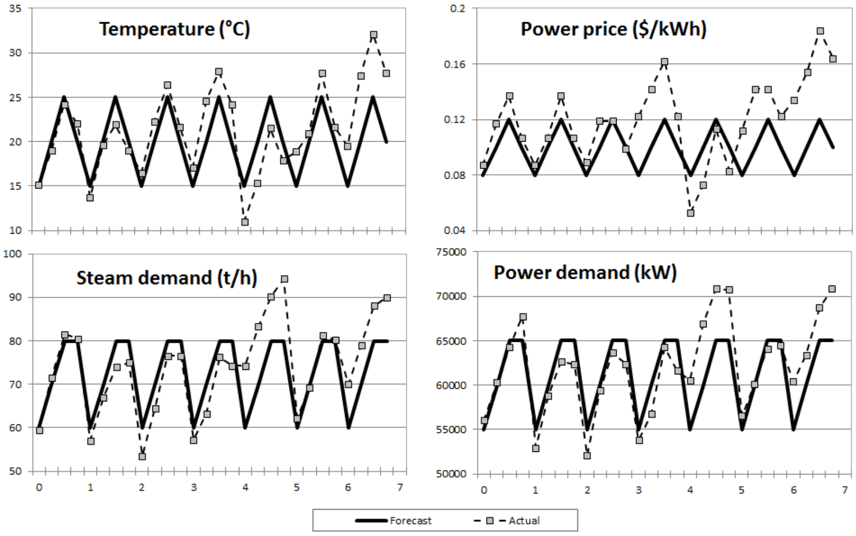

Fig. 3. Forecasted and actual values for ambient temperature, power price and steam and power demand.

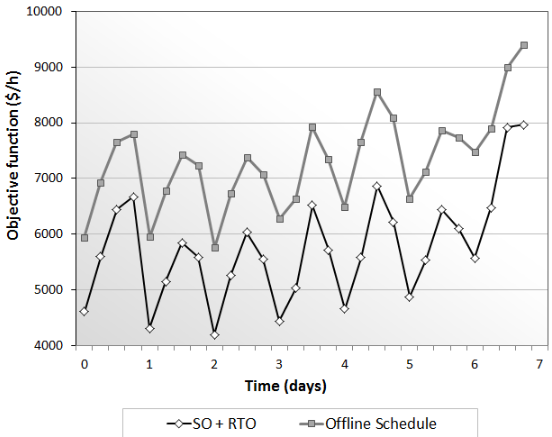

Fig. 4. Evolution of the objective function for the proposed SO+RTO strategy and the comparison case (offline scheduling optimization)

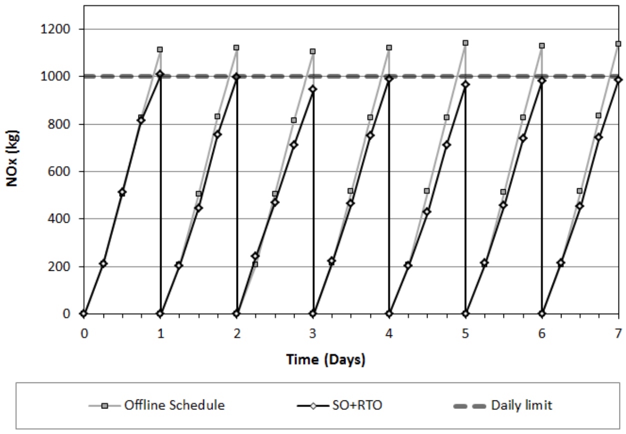

Fig. 5. Daily accumulated NOx production for offline scheduling optimization and SO+RTO strategies.

Figure 6a shows the steam vents for each period, with significant improvements when using the proposed methodology. This is directly related with the real-time management of backpressure turbines, shown in Figure 6b. Again, the real-time corrections of the model and the current and forecasted conditions lead to a reduction of the cost.

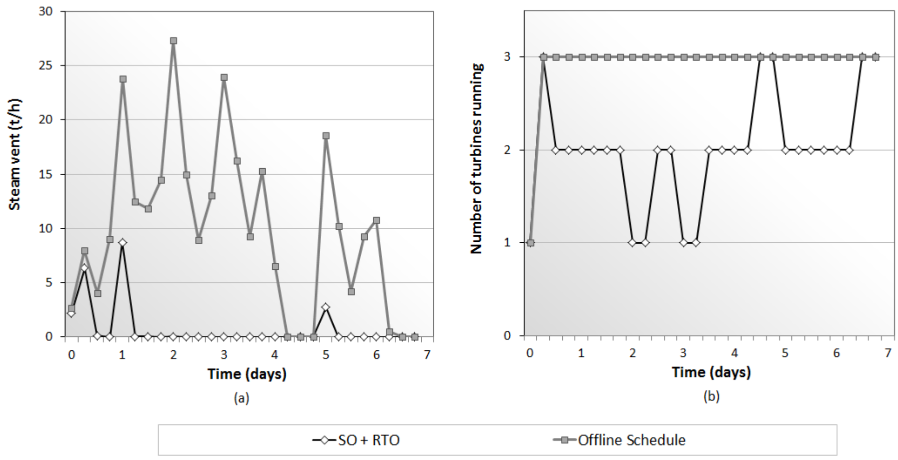

Fig. 6. (a) Steam vent and (b) Number of turbines operating for offline scheduling optimization and SO+RTO strategies.

5 Conclusions

A novel framework for integrating optimal scheduling and real-time optimization of continuous processes has been presented. It can be particularly useful for continuous improvement of the operation of combined heat and power systems. However, it can be generalized to other systems or processes.

This framework makes use of the higher number of degrees of freedom of an optimal scheduling formulation, as well as of the feedback properties and the lower computational cost of real-time optimization. Decision variables with long transition periods are fixed in the offline scheduling stage, while the remaining degrees of freedom are set in a multi-period RTO stage. The scheduling optimization can be performed offline and with low frequency, while RTO is performed using online measurements, which allows correcting the forecasted conditions and the model parameters. Except for the corrected parameters, the mixed-integer nonlinear models used in the scheduling and the RTO stage are the same.

A case study that optimizes 7-days a week operation of a combined heat and power system was used to illustrate the proposed methodology. A combined scheduling optimization+RTO strategy with a fixed horizon for the offline scheduling and a shrinking horizon for the RTO stage was implemented. The results show a significant improvement in operating cost reduction and constraint satisfaction, in comparison with the direct implementation of the offline scheduling results.

The MINLP problem solved in the real-time stage may present convergence problems that lead to an infeasible solution or to not obtaining a solution in the required (limited) time. A multiperiod MINLP real-time optimization system must have a strategy to recover from these problems, which will be the subject of future work.

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